Al-Khwārizmī's Contribution to Mathematics

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ABSTRACT

Mathematics in the Islamic opinion is considered as the gateway leading from the sensible to the intelligible world. The contribution of Muslims in the field of Mathematics is extremely remarkable. The Muslim mathematicians preserved and expanded the classics in Greek mathematics that the world otherwise would have lost. Muslims originally contributed in the field of algebra and it proved to be the greatest of their distinctive achievements in mathematics. In the ninth century, the Muslim mathematician, Al-Khwarizmi wrote his classical work on Algebra, Ḥisab Al-Jabr Wal Muqabalah (the science of cancellation and reduction). A Latin translation of this text became known in Europe under the title Al-Jabr. Thus, the Arabic word for reduction, Al-Jabr, became the word algebra. The Ḥisab Al-Jabr Wal-Muqabalah introduced the use of Indo-Arabic numerals that, over time, came to be known as algorithms. Here I will try to bring into limelight the contribution of Al-Khwarizmi in the field of mathematics and also focus on how much Al-Khwarizmi played an important role in European renaissance.

Keyword: *Al-Khwarizmi, Contribution, Algebra, Arithmetic, World.*

Introduction

Mathematics is a way of describing relationships between numbers and other measurable quantities. Through it can be expressed simple equations as well as interactions among the smallest particles and the farthest objects in the known universe. It deals with a group of related sciences, including algebra, geometry and calculus, concerned with the study of numbers, quantity shape and space and their interrelationship by using specialized notations. It is truly the language of science. Until the 17th century, arithmetic, algebra, and geometry were the only mathematical disciplines, and mathematics was virtually indistinguishable from science and philosophy. Developed by the ancient Greeks, these systems for investigating the world were preserved by Islamic scholars and passed on by Christian monks during the middle Ages.

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One of the other major points of Muslim contribution to mathematics was that they had also inherited a very rich and diverse body of knowledge from their predecessors: Babylonians, Greeks, Indians and Persians. From these diverse elements, they were to develop mathematics as a sophisticated and flexible tool in the pursuit of both theoretical and practical objectives.

Muslims performed regular studies of mathematics like all others and this begun during the reign of the second Abbasid caliph, Al-Maṇsur in the second half of the eighth century. During this period, Muslims carried their studies exclusively on mathematics. The investigations had been carried out until the end of the 11th century C.E. during which nearly all of the original and creative work had been done by the Muslim mathematicians and even in the 12th century, the Christians and Jews started the work of translation from Arabic into Latin and Hebrew and began to conduct research in this field.[1]

The city of Baghdad became a center of learning under the Muslim caliphate in 800 C.E. The great caliph, Al-Mamun, set scholars to work on translating all the great Greek texts into Arabic. This scientific knowledge was eventually transferred to Europe during the medieval ages. All religious groups Muslim, Christian, Jewish even Zoroastrian scholars had one thing in common that they translated the ancient writings into new scientific masterpieces. They all wrote in Arabic, the Muslim language. The caliphs not only supervised intelligently but many become patrons of learning and invited notable scholars to their courts. Numerous Hindu and Greek works in Mathematics translated into the Arabic language have been preserved. Later European scholars were able to retranslate them into Latin and other languages.

The contribution of some eminent Muslim mathematicians flourished during the Abbasid dynasty has been brought to lime light in the following lines.

Al-Khwārizmi's Contribution to Mathematics

Abū Abd Allāh Muḥ ammad ibn Mūsā al-Khwārizmi (780-850 CE) was the first outstanding Muslim mathematician who was born at Khawrizm, south of Aral Sea. The exact dates of his birth and death are not known, but it is established that he flourished under the caliphate of Al-Mamun at Baghdad during 813-833 C.E. and probably died around 850 C.E.[2]

Al-Khawrizmi was a mathematician, astronomer and a geographer. He was perhaps one of the greatest mathematicians who ever lived, as, in fact; he was the founder of several branches and basic concepts of mathematics. In the words of Hitti, he has influenced mathematical thought largely than any other medieval writers have.[3] David Eugene Smith and Louis Charles Karpinski describe Al-Khawrizmi as:

> ...the greatest master of the golden age of Baghdad, one of the first of the Muslim writers to collect the mathematical classics of both the east and the west, preserving them and finally passing them on to the awakening Europe. [4]

The renowned historian of mathematics, George Sarton realted, "al-Khwarizmi was one of the greatest scientists of his race and the greatest of his time and his personality was a well-developed scientific brilliance."[5]

Al-Khawrizmi's work on algebra was outstanding, as he not only initiated the subject in a systematic form but he also developed it to the extent of giving analytical solutions of linear and quadratic equations, which established him as the founder of Algebra. In the ninth century, Al-Khawrizmi wrote his classical work titled *Kitab Al-Mukhtasar Fi Ḥisab Al-Jabr Wal Muqabalah* (The Book of summary in the Process of Calculation for Compulsion and Equation). This is the first book known to contain this word '*Al-Jabr*'(still preserved in modern times as *algebra*) itself as title. The title consists of two words, *Al-Jabr Wal Muqabalah.* In this title, the word *Al Jabr* means transposing a quantity from one side of an equation to another, and *Muqabalah* signified the simplification of the resulting expressions. Thus, $x^2–2x = 5x+6$ passes by *Al-Jabr* into $x^2 = 5x+2x+6$; and this, by *Wal Muqabalah,* into $x^2 = 1x + 6$. The nearest English translation of which is "restoration" and "reduction".[6]

Some scholars maintain that this book of Al-Khawrizmi was influenced by the Indian mathematics but Florian Cajori rejects the opinion:

> The book was not influenced entirely by Indian sources because the Hindus had no rules like "restoration and reduction" as can be found in al Khwarizmi's Algebra. Besides, their language is pompous and encumbered by its verse form does not have the clarity exactness and scientific simplicity as that distinctly found in the languages of al Khwarizmi's text.[7]

In Al-Khawrizmi's era, algebra was a practical system for solving all kinds of problems like in cases of inheritance, contracts, surveying, tax collection, legacies, partition, lawsuits and trade and in all their dealings with one another, or where the measuring of lands, the digging of canals, geometrical computations, and other objects of various sorts and kinds were concerned. He dealt with the topic, which in Arabic is known as *'Ilm al-Fara'iḍ* (the science of the legal shares of the natural heirs). Gandz stated, Al-Khawrizmi's algebra:

> Al-Khwarizmi's algebra is regarded as the foundation and cornerstone of the sciences. In a sense, al-Khwarizmi is more entitled to be called "father of algebra" than Diaphantus because al-Khwarizmi is the first to teach algebra in the elementary form and for its own sake, Diophantus is primarily concerned with the theory of numbers.[8]

In Algebra important is the term 'root' that has its origin in the Arab language. 'Latin works translated from the Arabic have 'radix' as a common term. The Arabic word for *root* was used by Al-Khwarizmi to denote the first-degree term of a quadratic equation. Al-Khwarizmi, relates that "the following is an example of squares equal to root: a square is equal to 5 roots. The roots of square then is 5, and 25 form its square, which of course equals 5 of its roots."[9]

The Arabs' achievements in algebra were so great that it became known in contemporary academic circles as the product of the Arab genius, no wonder; by the word Muhammad bin Musa Al-Khwarizmi. The Arabs were also the first to use the algebraic symbols. They solved equations of the first second third and fourth degrees. They also succeeded in solving cubic equations by intersecting conics which in the words of Cajori: "was the greatest achievement of the Arabs in Algebra."[10]

Al-Khawarizimi and the West

The above-mentioned work of Al-Khwarizmi translated into Latin several times, under the title of *Liber Algorismi,* "book of the Khwarizmi"; became the source of the word "algorism."[11] In the twelfth century, Gerhard of Cremona and Robert of Chester translated the *Al-Jabr* of Al-Khwarizmi into Latin.[12] Robert of Chester translated it under the title of *Liber Algebrae Et Almucabala* in 1145 C.E. Western scholars used it until the sixteenth century.[13] This work of Al-Khwarizmi was used until sixteenth century as the principle mathematical

textbook in European universities and served to introduce into Europe, the science of algebra. Al-Khwarizmi's works were also responsible for the introduction of the Arabic numerals, called "algorisms" after him, into west. Among later mathematicians influenced by Al-Khwarizmi were 'Umar al-Khayyam, Leonardo Fibonacci of Pisa (after 1240 C.E.) and master Jacob of Florence, whose Italian treatise on mathematics, dated 1307 C.E., contains, as does one of Leonardo's works, the six types of quadratic equations given by the Muslim mathematicians.[14]According to David Eugene Smith, in the 16th century its name was found in English as *Algiebar* and *Almachabel*, and in various other forms, but was finally shortened to *algebra*. The word mean restoration and opposition and of the clearest explanations of their use is given by Baha' al-Din (1600 C.E.) in his *Khulaṣat Al-Ḥisab* (essence of arithmetic).[15] This book of Al-Khwarizmi on Algebra may be regarded as the foundation and cornerstone of mathematics and many historians of mathematics (1979 C.E.) Boyer (1905 C.E.), Gand (1936 C.E.); Rasheed (1980 C.E.) felt that Al-Khwarizmi, deserved to be called as the "Father of Algebra" more than anyone else is.[16]

Al-Khwarizmi's work on arithmetic was instrumental in disseminating the Arabic numeral system and in projecting algebra as an important branch of mathematics in the European world. Al-Khwarizmi's arithmetical work possibly entitled *Kitab al-Jam Wal Tafriq Bi Ḥisab al-Hind* (Book of Addition and Subtraction according to the Hindu Calculation) was written after he had composed his other reputed works on Algebra. Its original Arabic version was lost but the Latin translation was discovered in 1857 C.E. at the library of the University of Cambridge. It is believed that this was a copy of Al-Khwarizmi's arithmetical treatise that was translated presumably by Abelard of Bath during the twelfth century. B. Boncompagui first published it under the title *Algoritmi De Numero In Dorum* in Rome in 1857 C.E. and later Kurt Vogel under the title *Alchwarizmi's Algorithmus* at Aalen in 1963 C.E. Al-Khwarizmi's treatise is the earliest known textbook written on decimal system. Although elementary, it represents a milestone in the development of mathematics and science.[17] This translated version of Al-Khwarizmi's text found its way to Italy, Spain, and England. However, subsequent Latin translations of his work become known as *Alchwarizmi*, *Al-Karismi, Algoritmi, Algorismi*, *Algorithm* etc. Thus, Al-Khwarizmi left his name to the history of mathematics in the form of *algorism*, the old word for arithmetic.[18]European scholars linked Al-Khwarizmi to the new arithmetic that finally became the scheme of numeration, making use of the

Arabic numerals called *algorithm*. This word originates from a corruption of the name Al-Khwarizmi that now means a definite procedure for calculating something.[19]

Al-Khwarizmi explained the use of zero, a numeral of fundamental importance developed by Arabs. After its introduction by Al-Khwarizmi the symbol zero was known and used for at least 250 years in the Islamic world before the west came to know it. It is said that Hindus originated *Shunya (zero)* but it took Europe at least two hundred and fifty years to accept and acknowledge the 'zero' as a gift from Muslims.[20] American mathematics Professor Karl J. Smith indicated in his textbook, *The Nature of Mathematics*, that the "zero" is a gift from the Arabs.[21] In Latin, it was simultaneously translated as *cipher, ziffer, zephillen, zenero, cincro* and many others. In Italian, it was called *zenero, cenero* and *zephiro.* Since the fourteenth century, zero has been the term used as shown in the records of 1491 C.E. by Calnadri and of 1494 C.E. by Luca Pacisli. Up until the 17th and 18th century C.E., mathematicians such as Adrian Metiers (1611 C.E.), Herigone (1634 C.E.), Cavalieri (1643 C.E.) and Euler (1783 C.E.) still used *cipher* (a transliteration of the Arabic *ṣifar)* to denote zero.[22] The zero has a great important in arithmetic. Without zero it is not possible to indicate the figures like tens, hundreds etc. It becomes necessary to use a table (named abacus) with columns of units tens, hundreds etc. to keep each figure in its place if zero is not used. The Arabic word for zero is *ṣifar* meaning empty or nil, and the Latin word for zero is of Arabic origin.[23]

Thus, Al-Khwarizmi's treatises on arithmetic and Algebra became the principle sources of mathematical knowledge for several centuries in both the East and the West. In addition to introducing the Indian system of numerals (now generally known as Arabic numerals), he developed at length several arithmetical procedures, including operations on fractions.

Al-Khwarizmi's algebra also contained some geometrical ideas, according to Florian Cajori, he is not only to have given the theorem of the right triangle when it is isosceles, but also calculated the areas of the triangle, parallelogram, and circle. In addition, one chapter in Al-Khwarizmi's *Al-Jabr* on menstruation dealt only with geometry and is called *Bab al-Misaha* (chapter on measurements of areas).[24]

Al-Khwarizmi was one of the first to compute astronomical and trigonometrical tables. Included in his work on trigonometry, were his one hundred tables of sine and cotangent values.[25] Thus, Al-Khwarizmi was a mathematician of great acumen and high caliber whose contribution remains unparalleled in this significant field of study.

Conclusion

The Muslims contributed more, than others did, to the field of mathematics, which is considered as the foundation of modern civilization. Al-Khwarizmi (780-850), the most genius and a mathematician of high repute during Abbasid Baghdad, created modern algebra and made brilliant contributions in the field of mathematics. His work translated many European and other languages that is also use in today life.

End Notes and Reference

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